BUAD 340 Principles of Finance Fall 2018/Dr. Minor Time Value of Money Handout 181001

Perpetuities

A **perpetuity** is an annuity that continues forever or has no maturity. For example, a dividend stream on a share of preferred stock. There are two basic types of perpetuities:

- **Growing perpetuity** in which cash flows grow at a constant rate, g, from period to period.
- **Level perpetuity** in which the payments are constant rate from period to period.

Even if the cash flows are infinite, present values can be finite if the discount rate is higher than the growth rate.

Present Value of a Level Perpetuity

with n=infinity; = PMT/ i

- PMT = level (constant) payment per period.
- I = rate per period.

- Example 6.6 What is the present value of $600 perpetuity at 7% discount rate?
  - PV=600/0.07=**8751.43.**
- If you decide to rent an apartment with a fixed rent of $2,000 per month and live there forever (subletting it to your children after you die), how much is this apartment worth if the mortgage rate is 6% per year (Ignore tax, liquidity and other concerns).
  - The present value of paying $2000 per month forever at 6% rate per year is: PV=2000/(0.06/12)=**400,000.**
  - 200 times your rent is about the house value

- Checkpoint 6.4. The Present Value of a Level Perpetuity
  - What is the present value of a perpetuity of $500 paid annually discounted back to the present at 8 percent? **$6,250**
  - What is the present value of stream of payments equal to $90,000 paid annually and discounted back to the present at 9 percent? **$1,000,000**

Present Value of a Growing Perpetuity

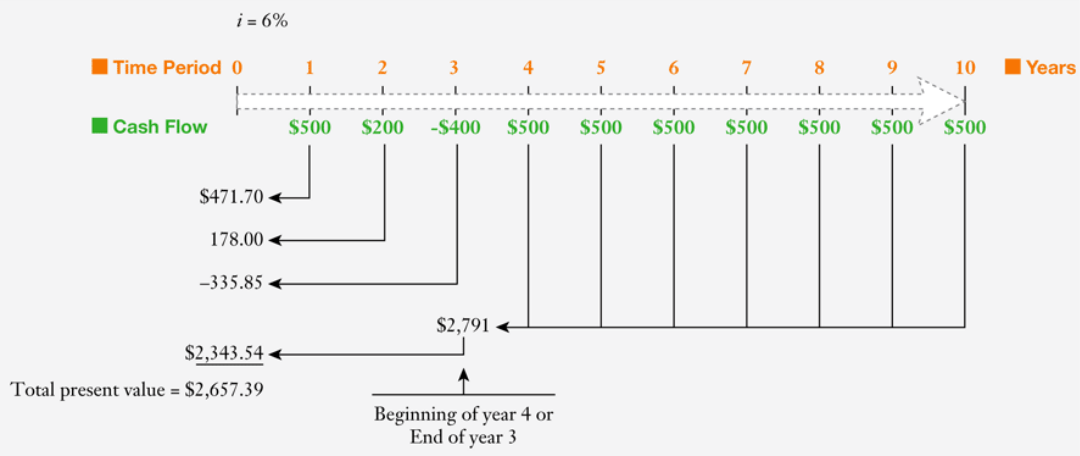
- In growing perpetuities, the periodic cash flows grow at a constant rate each period.
- The present value of a growing perpetuity can be calculated using a simple mathematical equation:
  - **i -- rate per period, g—growth per period,**
  - **PMTperiod 1 – payment at the end of the first period.**

- Checkpoint 6.5. The Present Value of a Growing Perpetuity
  - What is the present value of a perpetuity stream of cash flows that pays $500 at the end of year one but grows at a rate of 4% per year indefinitely? The rate of interest used to discount the cash flows is 8%. PV=500/(.08-.04)=500/.04=**12,500**
  - What if the growth rate is 6%? PV=500/(.08-.06)=500/.02=**25,000**
  - What if the growth rate is 9%? When growth rate is faster than discount rate, the present value is infinite. **You can no longer use the formula.**

Complex Cash Flow Streams

The cash flows streams in the business world may not always involve one type of cash flows. The cash flows may have a mixed pattern. For example, different cash flow amounts mixed in with annuities.

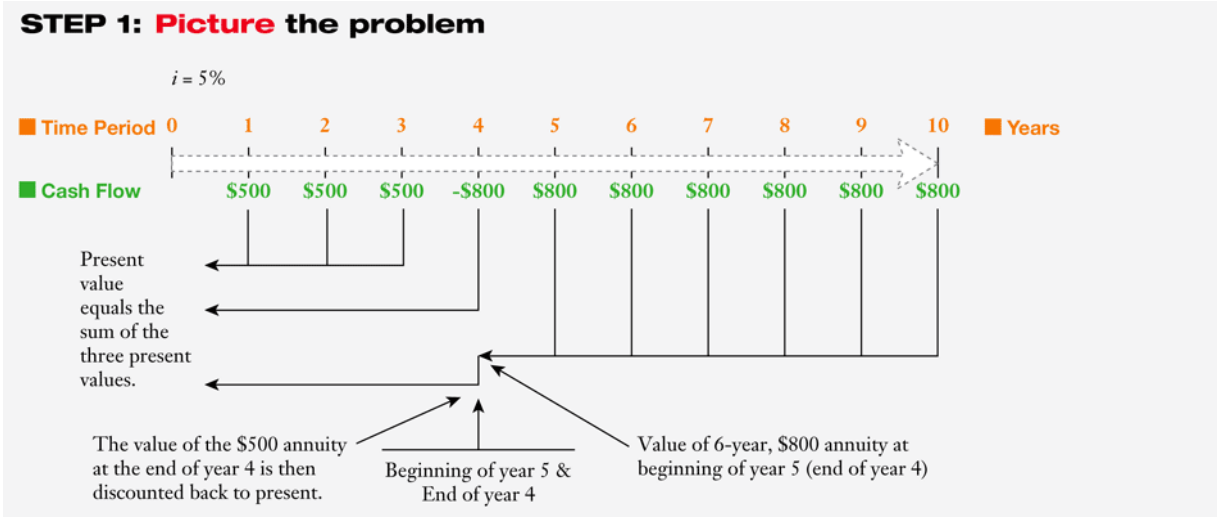
- For example, figure 6-4 summarizes the cash flows for Marriott.



- In this case, we can find the present value of the project by summing up all the individual cash flows by proceeding in four steps:
  - Find the present value of individual cash flows in years 1, 2, and 3.
  - Find the present value of ordinary annuity cash flow stream from years 4 through 10.
  - Discount the present value of ordinary annuity (step 2) back three years to the present.
  - Add present values from step 1 and step 3.

- Checkpoint 6.6. The Present Value of a Complex Cash Flow Stream

What is the present value of cash flows of $500 at the end of years through 3, a cash flow of a negative $800 at the end of year 4, and cash flows of $800 at the end of years 5 through 10 if the appropriate discount rate is 5%?



- PV of 3x5000=500*[(1-1.05^{-3})/.05]=1361.62
- PV of (-800)=-800/1.05^{4}=-658.16
- Year 4 value of 6x800= 800*[(1-1.05^{-6})/.05]=4060.55
  - PV=4060.55/1.054=3340.63
- Total PV=1361.62-658.16+3340.63=**4044.09**

Checkpoint 6.6: What is the present value of cash flows of $300 at the end of years 1 through 5, a cash flow of negative $600 at the end of year 6, and cash flows of $800 at the end of years 7-10 if the appropriate discount rate is 10%?

- Group the cash flow in to three types, all with i=10%
  - $300 from year 1 to 5
  - -$600 at year 6
  - $800 from year 7-10
- Find PV for each group:
  - PV=300[(1-1.1-5)/0.1]=1137.24
  - PV=-600/1.16=-338.68
  - PV={800[(1-1.1-4)/0.1]}/1.16=1431.44 (two steps here)
- Total PV=**2300.00**