

BUAD 340 Principles of Finance Fall 2018/Dr. Minor Time Value of Money Handout 180926

- FV_n = FV of annuity at the end of n th period.
- PMT = annuity payment deposited or received at the end of each period.
- i = interest rate per period
- n = number of periods for which annuity will last.

Ordinary Annuities

An annuity is a series of equal dollar payments that are made at the end of equidistant points in time such as monthly, quarterly, or annually over a finite period of time. If payments are made at the end of each period, the annuity is referred to as ordinary annuity

Ordinary Annuities: Solving for FV

$$FV_n = PMT \left[\frac{(1+i)^n - 1}{i} \right]$$

- Example 6.1 How much money will you accumulate by the end of year 10 if you deposit \$3,000 each for the next ten years in a savings account that earns 5% per year? **\$37,733.68**
 - $FV = \$3000 \{ [(1+.05)^{10} - 1] \div (.05) \}$
 $= \$3,000 \{ [0.63] \div (.05) \}$
 $= \$3,000 \{ 12.58 \} = \mathbf{\$37,733.68}$

Ordinary Annuities: Solving for PMT

$$PMT = FV_n / \left[\frac{(1+i)^n - 1}{i} \right]$$

- Example 6.2: Suppose you would like to have \$25,000 saved 6 years from now to pay towards your down payment on a new house. If you are going to make equal annual end-of-year payments to an investment account that pays 7%, how big do these annual payments need to be? **\$3494.89**
- How much must you deposit in a savings account earning 8% interest in order to accumulate \$5,000 at the end of 10 years? **\$345.15**
- If you can earn 12% on your investments, and you would like to accumulate \$100,000 for your child's education at the end of 18 years, how much must you invest annually to reach your goal? **\$1793.73**

Ordinary Annuities: Present Value

$$PV = FV_n / (1+i)^n, \text{ where } FV_n = PMT \left[\frac{(1+i)^n - 1}{i} \right]$$

- Example: What is the value today or lump sum equivalent of receiving \$3,000 every year for the next 30 years if the interest rate is 5%? $FV=199,316.54$. $PV=\mathbf{\$46,117.35}$
- Checkpoint 6.2 Your grandmother has offered to give you \$1,000 per year for the next 10 years. What is the present value of this 10-year, \$1,000 annuity discounted back to the present at 5%? **\$7721.73**
- Checkpoint 6.2: What is the present value of an annuity of \$10,000 to be received at the end of each year for 10 years given a 10 percent discount rate? **\$61,445.67**

Amortized Loans

$$PMT = PV * i / [1 - 1/(1+i)^n]$$

An **amortized loan** is a loan paid off in equal payments – consequently, the loan payments are an annuity.

Examples: Home mortgage loans, Auto loans

- In an amortized loan, the *present value* can be thought of as the amount borrowed, n is the number of periods the loan lasts for, i is the interest rate per period, and *payment* is the loan payment that is made.
- Example 6.5 Suppose you plan to get a \$9,000 loan from a furniture dealer at 18% annual interest with annual payments that you will pay off in over five years. What will your annual payments be on this loan?
- $PMT = PV * i / [1 - 1/(1+i)^n] = \mathbf{\$2,878.00}$

- The Loan Amortization Schedule on www.michaelominor.com.
 - Size of each payment remains the same.
 - However, interest payment declines each year as the amount owed declines & more of the principal is repaid.
- Many loans such as auto and home loans require monthly payments. This requires converting n to number of months and computing the monthly interest rate.
- Example 6.6 You have just found the perfect home. However, in order to buy it, you will need to take out a \$300,000, 30-year mortgage at an annual rate of 6 percent. What will your monthly mortgage payments be?
 - $n=30*12=360$. $i=6\%/12=0.5\%$
 - $PMT=300000*.005/[(1-1/(1+.005)^360)] = \mathbf{\$1798.65}$

Determining the Outstanding Balance of a Loan

- Checkpoint 6.3. Let's say that exactly ten years ago you took out a \$200,000, 30-year mortgage with an annual interest rate of 9 percent and monthly payments of \$1,609.25. But since you took out that loan, interest rates have dropped. You now have the opportunity to refinance your loan at an annual rate of 7 percent over 20 years. You need to know what the outstanding balance on your current loan is so you can take out a lower-interest-rate loan and pay it off. If you just made the 120th payment and have 240 payments remaining, what's your current loan balance? What will be your new monthly payment if you can do the refinancing?
 - Double check the payment: $PV=200,000$, $n=360$, $i=0.09/12=0.0075$.
 - $PMT=PV/[(1-1.0075^{-360})/0.0075]=\mathbf{\$1609.25}$
 - The remaining principal can be computed as the present value of the remaining payments under the existing interest rate (9%).
 - Remaining balance= $PV = 1609.245[(1-(1.0075)^{-240})/(0.0075)]=\mathbf{\$178,859.49}$
 - Now we can compute the new monthly payment on the remaining balance with a new rate $i=0.05/12= 0.00583$
 - $PMT=178859.49/[(1-1.00583^{-240})/0.00583]= \mathbf{\$1,386.69}$
 - A monthly saving of $\mathbf{\$222.55}$ ($=1609.25-1386.69$)

Checkpoint 6.3: Let's assume you took out a \$300,000, 30-year mortgage with an annual interest rate of 8%, and monthly payment of \$2,201.29. Since you have made 15 years' worth of payments, there are 180 monthly payments left before your mortgage will be totally paid off. How much do you still owe on your mortgage? Hint: The remaining balance is essentially the present value of remaining payments under the existing rate. Verify the answer: $\mathbf{\$230,344.29}$

Annuities Due

FV or PV (annuity due) = (FV or PV (ordinary annuity) x (1+i))

Annuity due is an annuity in which all the cash flows occur at the beginning of the period. For example, rent payments on apartments are typically annuity due as rent is paid at the beginning of the month.

- Computation of future/present value of an annuity due requires compounding the cash flows for one additional period, beyond an ordinary annuity.
- Example 6.1 where we calculated the future value of 10-year ordinary annuity of \$3,000 earning 5% to be \$37,734. What will be the future value if the deposits of \$3,000 were made at the beginning of the year i.e. the cash flows were annuity due?
 - Just compound the future value for the ordinary annuity for one more period: $FV=37734 \times 1.05=\mathbf{\$39,620.70}$
- Checkpoint 6.2 where we computed the PV of 10-year ordinary annuity of \$10,000 at a 10% discount rate to be equal to \$61,446. What will be the present value if \$10,000 is received at the beginning of each year i.e. the cash flows were annuity due?
 - Just compound the PV of the ordinary annuity for one more period: $PV=61446 \times 1.1=\mathbf{\$67,590.60}$